INJECTIVE CONTINUOUS REDUCTION ON BOR(ω^ω)

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Given two subsets of the Baire space A and B, a *continuous injective reduction* from A to B is a continuous injective function $f : \omega^\omega \to \omega^\omega$ satisfying $f^{-1}[B] = A$, We say that A *injectively Wadge reduces* to B, and we denote it by $A \leq_i B$. The structure $(\mathrm{BOR}(\omega^\omega), \leq_i)$ is a quasi-order and as usual with quasi-orders, there is an induced equivalence relation: for $A, B \subseteq \omega^\omega$ we write $A \equiv^i_{\mathrm{W}} B$ if both $B \leq_i A$ and $A \leq_i B$ hold. We call *injective Wadge degree* of $A \subseteq \omega^\omega$ the $\equiv^i_{\mathrm{W}}$-equivalence class of A. Injective Wadge reduction induces a partial order on injective Wadge degrees that we call the *injective Wadge hierarchy* on $\mathrm{BOR}(\omega^\omega)$. Work in progress is to prove that injective Wadge hierarchy on the Borel subsets of the Baire space is bqo. We will discuss the partial positive answers. This is joint work with Raphaël Carroy.